Exercise 06

A cascade of repressors

Consider a cascade of three repressors:

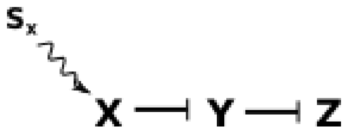

Figure 1: Cascade of three repressors. Signal S_x is keeping X in its inactive form and prevents the production of Y.

Protein X is initially present in the cell in its inactive form. Protein Z is present in the cell at its steady state concentration, and is continuously produced at rate β. At $t = 0$, the input signal of X, S_x is removed. X is rapidly converted to its active form, X^*, and binds to the promoter of gene Y. As a result, production of protein Y begins. When Y levels increase above threshold K_Y, gene Z is repressed. All proteins have the same degradation/dilution rate α.

a) *Sketch* the concentration of Y and Z as a function of time.

b) Show that the response time of Y, with respect to the addition of S_x, is:

$$t_{Y_{1/2}} = \frac{ln(2)}{\alpha}$$

c) Show that the time at which production of Z stops, t_{YZ}, is:

$$t_{YZ} = \frac{1}{\alpha} \cdot ln\left(\frac{Y_{st}}{Y_{st} - K_Y}\right)$$

d) What is the response time of Z, with respect to the addition of S_x?

e) What is the effect of placing Z at the end of a cascade, as here, compared with placing Z under simple repression? Why might this effect be biologically useful?